Longitudinal Dynamics with Finite Betatron Amplitude

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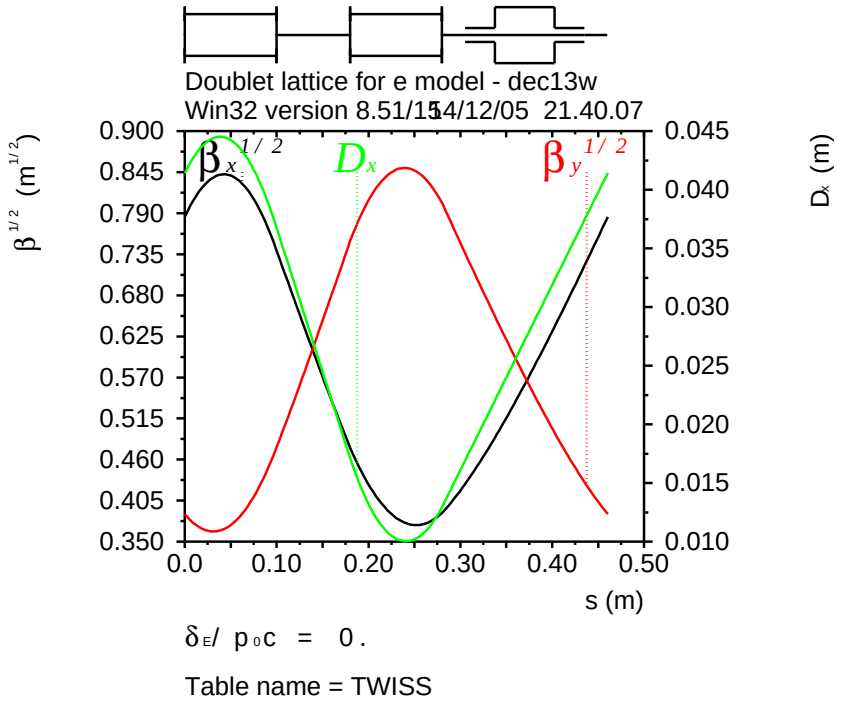
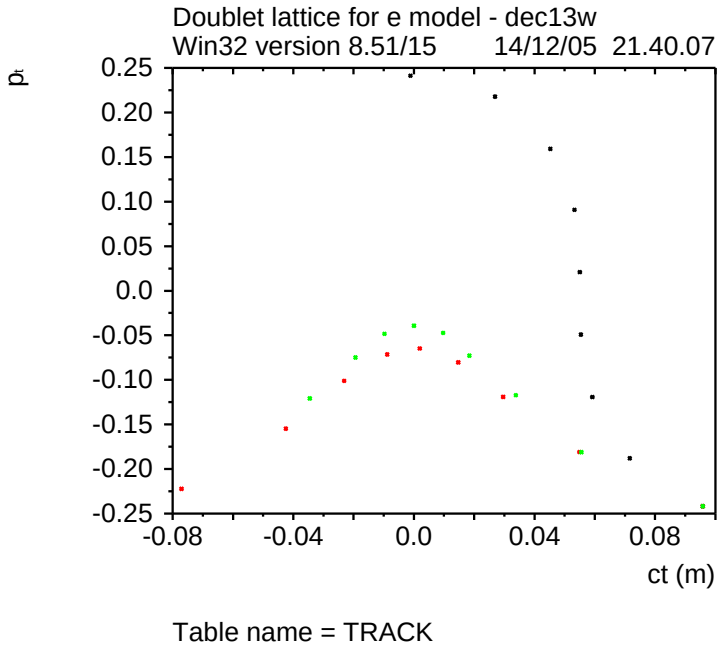
My WWW home directory:
`http://keil.home.cern.ch/keil/`
`MuMu/Doc/ElectronModel/20Dec05/talk.pdf`

Layout and Orbit Functions

- Any lattice and any program will show longer time of flight for particles with finite betatron amplitude, if one looks for it
- As example use electron model lattice that accelerates between $-0.25 \leq \delta p/p \leq +0.25$, less than official model, and has single zero of time of flight at 15 MeV
- Calculation with mad8, using reference energy 15 MeV



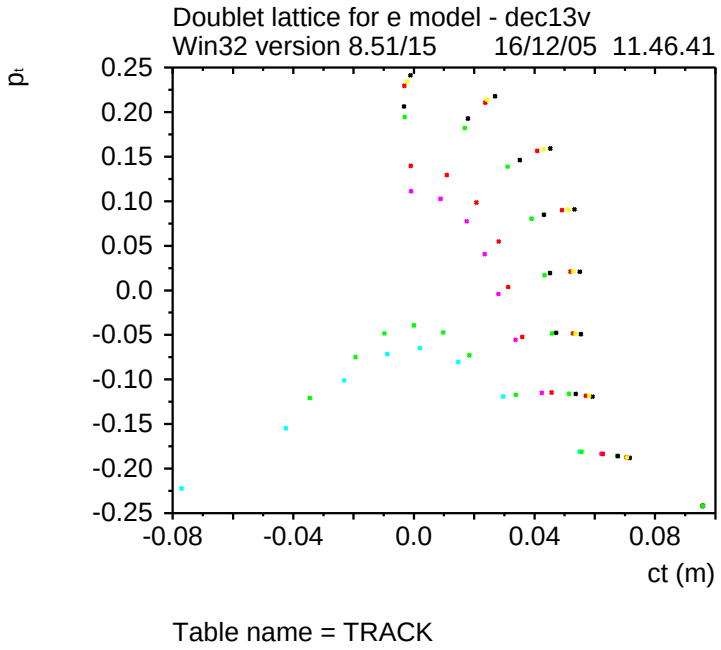
Trajectories in Longitudinal Phase Space I



- Abscissa is ct counted from last zero crossing of RF wave
- My sign of ct is opposite to Machida's
- Ordinate is $\delta p/p$
- Black trajectory has vanishing betatron amplitude, and acceleration from $\delta p/p \approx -0.25$ to $\delta p/p \approx +0.25$ in 8 turns by design

- Red and green trajectories start with $+4$ mm and -4 mm offset from closed orbit
- Red and green trajectories move more slowly in ct and are accelerated much less
- RF cavity voltage $V_{\mathrm{RF}} = 35.06$ kV

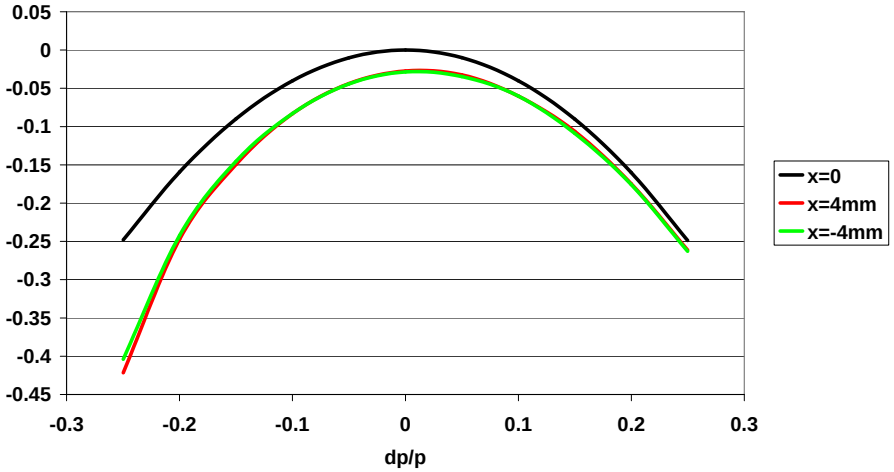
Trajectories in Longitudinal Phase Space II



- Abscissa is ct counted from last zero crossing of RF wave
- My sign of ct is opposite to Machida's
- Ordinate is $\delta p/p$
- Black trajectory has vanishing betatron amplitude, and acceleration from $\delta p/p \approx -0.25$ to $\delta p/p \approx +0.25$ in 8 turns by design

- Coloured trajectories start with horizontal offsets from closed orbit between $+4$ mm and -4 mm in steps of 1 mm
- With increasing horizontal offset from closed orbit rajectories move more slowly in ct and are accelerated less
- Limit of horizontal offset $|x| \approx 2 \ldots 3$ mm

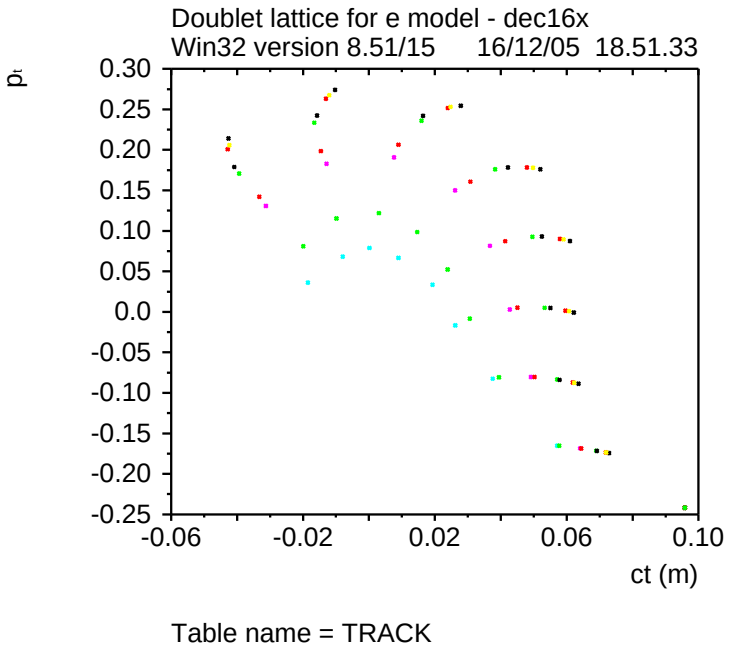
Time of Flight vs. Horizontal Offset



- Abscissa is $\delta p/p$
- Ordinate is difference in time of flight ct in metres between actual and reference particle in ring, taken for 8 turns to average out betatron oscillations
- Particles with $ct < 0$ take longer than reference particle

- Black trajectory has vanishing betatron amplitude and symmetrical shape by design, total spread in $ct \approx 0.25$ m
- Red and green trajectories start with horizontal offsets $x = \pm 4$ mm, almost coincide, are asymmetrical, and have $ct < 0$ at all $\delta p/p$
- Asymmetry probably caused by drop of D'_x with $\delta p/p$ due to reduced focusing
- Coefficient η_1 of quadratic variation of ct increases by factor 1.26
- RF voltage $V_{\mathrm{RF}} \propto \eta_1$ according to theory presented on 23 Feb 2005

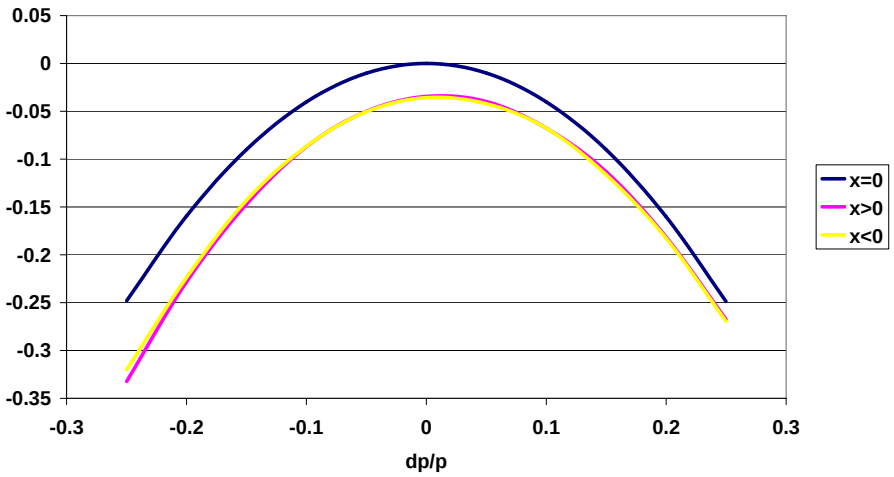
Acceleration with Higher RF Voltage



- RF voltage $V_{\mathrm{RF}} = 44.34$ kV
- Black trajectory with horizontal offset $x = 0$ reaches $\delta p/p > 0.25$
- Maximum $\delta p/p$ reached in 7 instead of 8 turns
- Trajectories with horizontal offsets $x = \pm 4$ mm reach $\delta p/p > 0$

- Large spread in $\delta p/p$ and small spread in ct due to betatron oscillations
- Banana shaped distortion
- Larger transverse acceptance than previous case
- Poor quantitative agreement with theory

Time of Flight at Constant Normalised Emittance



- Ordinate is difference in time of flight ct in metres between actual and reference particle in ring, taken for 208 cells or 8 turns to average out betatron oscillations
- Particles with $ct < 0$ take longer than reference particle

- Blue trajectory has vanishing betatron amplitude and symmetrical shape by design, total spread in $ct \approx 0.25$ m
- Red and yellow trajectories start with horizontal offsets x corresponding to normalised emittance $\varepsilon_n = 5$ mm, almost coincide, are asymmetrical, and have $ct < 0$ at all $\delta p/p$
- Asymmetry probably caused by drop of D'_x with $\delta p/p$ due to reduced focusing
- Coefficient η_1 of quadratic variation of ct increases by factor 1.26